$d=5,\ \mathcal{N}=1$:

SCA: $F(4)$

→ R-sym. : $SU(2)_R$

→ L-sym. : $SO(5) = Sp(4) \longrightarrow j_1, j_2 \in \mathbb{Z}_{\geq 0}$

i.e. $[1,0]$ and $[0,1]$ denote spinor $\mathbf{4}$ and vector $\mathbf{5}$ of $SO(5)$

→ labeling : $[j_1, j_2]_\Delta^{(R)}$, $j_1, j_2, R \in \mathbb{Z}_{\geq 0}$

→ Q-supercharges: $Q \in [1,0]_{1/2}^{(1)}$, $N_Q = 8$

Shortening conditions:

Name	Primary	Unitarity Bound	Null state
L	$[j_1, j_2]_\Delta^{(R)}$	$\Delta > j_1 + j_2 + \frac{3}{2}R + 4$	–
A_1	$[j_1, j_2]_\Delta^{(R)}, j_1 \geq 1$	$\Delta = j_1 + j_2 + \frac{3}{2}R + 4$	$[j_1 - 1, j_2]_{\Delta+1/2}^{(R+1)}$
A_2	$[0, j_2]_\Delta^{(R)}, j_2 \geq 1$	$\Delta = j_2 + \frac{3}{2}R + 4$	$[0, j_2 - 1]_{\Delta+1}^{(R+2)}$
A_4	$[0,0]_\Delta^{(R)}$	$\Delta = \frac{3}{2}R + 4$	$[0,0]_{\Delta+2}^{(R+4)}$
B_1	$[0, j_2]_\Delta^{(R)}, j_2 \geq 1$	$\Delta = j_2 + \frac{3}{2}R + 3$	$[1, j_2]_{\Delta+1/2}^{(R+1)}$
B_2	$[0,0]_\Delta^{(R)}$	$\Delta = \frac{3}{2}R + 3$	$[0,0]_{\Delta+1}^{(R+2)}$
C_1	$[0,0]_\Delta^{(R)}$	$\Delta = \frac{3}{2}R$	$[1,0]_{\Delta+1/2}^{(R+1)}$

examples:

- $C_1[0,0]^{(1)}_{3/2}$: free hypermultiplet
- $C_1[0,0]^{(2)}_{3}$: flavor current multiplet
- $B_2[0,0]^{(0)}_{3}$: stress-tensor multiplet

supersymmetric deformations:

Primary	Deformation $\delta\mathcal{L}$	Comments
$C_1\left\{\begin{matrix}(2)\\ \Delta_{\mathcal{O}}=3\end{matrix}\right\}$	$Q^2\mathcal{O}\in\left\{\begin{matrix}(0)\\ \Delta=4\end{matrix}\right\}$	Flavor Current } rel.
$C_1\left\{\begin{matrix}(R+4)\\ \Delta_{\mathcal{O}}=6+\frac{3}{2}R\end{matrix}\right\}$	$Q^4\mathcal{O}\in\left\{\begin{matrix}(R)\\ \Delta=8+\frac{3}{2}R\end{matrix}\right\}$	F-term } irrelevant
$L\left\{\begin{matrix}(R)\\ \Delta_{\mathcal{O}}>4+\frac{3}{2}R\end{matrix}\right\}$	$Q^8\mathcal{O}\in\left\{\begin{matrix}(R)\\ \Delta>8+\frac{3}{2}R\end{matrix}\right\}$	D-term

$d=6$, $\mathcal{N}=1$:

SCA: $osp(8|2)$

- → R-sym.: $sp(2)_R \simeq SU(2)_R$
- – L-sym.: $SO(6) = SU(4)$

$$[j_1, j_2, j_3]\ ,\quad j_1, j_2, j_3 \in \mathbb{Z}_{\geq 0}$$

$[1,0,0]$ and $[0,0,1]$ are left- and right-handed chiral spinors 4, $4'$ of $SO(6)$

$[0,1,0]$ is vector rep. $\mathbf{6}$ of $SO(6)$.

$Q \in [1,0,0]^{(1)}_{1/2}$, $\quad N_Q = 8$

Supersymmetric deformations:

Primary $\mathcal{O}$	Deformation $\delta\mathcal{L}$	Comments
$D_1 \begin{Bmatrix} (R+4) \\ \Delta_{\mathcal{O}} = 8+2R \end{Bmatrix}$	$Q^4\mathcal{O} \in \begin{Bmatrix} (R) \\ \Delta = 10+2R \end{Bmatrix}$	F-term
$L \begin{Bmatrix} (R) \\ \Delta_{\mathcal{O}} > 6+2R \end{Bmatrix}$	$Q^8\mathcal{O} \in \begin{Bmatrix} (R) \\ \Delta > 10+2R \end{Bmatrix}$	D-term

→ neither relevant nor marginal defs

→ only possible supersymmetric RG-flows: moving onto moduli space of vacua

<u>$d=6$, $\mathcal{N}=2$</u>:

SCA: $\quad Osp(8|4)$

→ R-sym.: $Sp(4)_R$

(R_1, R_2)

$(1,0) \to \mathbf{4}$ of $sp(4)$, $\quad (0,1) \to \mathbf{5}$

$Q \in [1,0,0]^{(1,0)}_{1/2}$, $\quad N_Q = 16$

Supersymmetric deformations:

Primary $\mathcal{O}$	Deformation $\delta\mathcal{L}$	Comments
$D_1 \left\{ \begin{matrix} (0, R_2+4) \\ \Delta_{\mathcal{O}} = 8+2R_2 \end{matrix} \right\}$	$Q^8\mathcal{O} \in \left\{ \begin{matrix} (0, R_2) \\ \Delta = 12+2R_2 \end{matrix} \right\}$	F-term
$D_1 \left\{ \begin{matrix} (R_1+4, R_2) \\ \Delta_{\mathcal{O}} = 8+2R_2 \end{matrix} \right\}$	$Q^{12}\mathcal{O} \in \left\{ \begin{matrix} (R_1, R_2) \\ \Delta = 14+2(R_1+R_2) \end{matrix} \right\}$	–
$L \left\{ \begin{matrix} (R_1, R_2) \\ \Delta_{\mathcal{O}} > 6+2(R_1+R_2) \end{matrix} \right\}$	$Q^{16}\mathcal{O} \in \left\{ \begin{matrix} (R_1, R_2) \\ \Delta > 14+2(R_1+R_2) \end{matrix} \right\}$	D-term

$\longrightarrow$ no relevant or marginal deformations

§ 2.3 Deformations related to conserved currents

1) Flavor current multiplets:

Lorentz-invariant defs residing in same multiplet as j^a_μ, a is adjoint flavor index.

$\rightarrow$ relevant defs, $\Delta = d-1 < d$

general form in Lagrangian: $m_a (\bar{\psi}\psi)^a$

(fermion mass term)

$\rightarrow$ denote collectively by $\mathcal{M}^a_{fl}$

Flavor mass defs can occur in following theories:

- 3d $2 \leq \mathcal{N} \leq 4$ SCFTs

$\mathcal{N}=2$: $\mathcal{M}^a_{fl} \in [0]^{(0)}_2$ neutral under $U(1)_R$

resides at level 2 in $A_2\bar{A}_2[0]^{(0)}_1$

flavor current multiplet.

$\mathcal{N}=4$: two different defs $(\mathcal{M}^a_{fl})^{(ij)} \in [0]^{(2;0)}_1$

and $(\mathcal{M}'^a_{fl})^{(i'j')} \in [0]^{(0;2)}_2$

(triplets under $SU(2)_R$ and $SU(2)'_R$)

reside at level 2 in $B_1[0]^{(0;2)}_1$ and $B_1[0]^{(2;0)}$ (exchanged under M)

- 4d $\mathcal{N}=2$ SCFTs admit complex mass defs $\mathcal{M}^a_{ff}$ and $\overline{\mathcal{M}}^a_{ff}$:

  $\mathcal{M}^a_{ff} \in [0;0]_3^{(0;2)}$ and $\overline{\mathcal{M}}^a_{ff} \in [0;0]_3^{(0;-2)}$

  (reside at level 2 in $\mathcal{B}, \overline{\mathcal{B}}_1 [0;0]_2^{(2;0)}$)

- 5d $\mathcal{N}=1$ SCFTs admit real flavor mass defs $\mathcal{M}^a_{ff}$, neutral under $SU(2)_R$,

  $\mathcal{M}^a_{ff} \in [0,0]_4^{(0)}$, reside at level 2 in $C_1[0,0]_3^{(2)}$ multiplet

Schematically, we can write

$$\delta \mathcal{L}_{ff} = m_{a,I}\, \mathcal{M}^{a,I}_{ff} + \mathcal{O}(m^2) = m_{a,I} (QJ^a)^I + \mathcal{O}(m^2)$$

$m_{a,I}$ can be interpreted as residing in non-dynamical vector multiplets containing background flavor gauge fields.

Intuitive picture:

6d (1,0) with $N_Q = 8$

$V_{6d} = (A_\mu, \psi)$ (no scalar in 6d Vector-mult.)

$\rightarrow$ no flavor mass defs.

$\downarrow$

5d $\mathcal{N}=1$

$V_{5d} = (\phi, A_\mu, \psi)$ $\phi = A_5$ is 5d scalar

$\rightarrow$ 1 mass def.

$\downarrow$

4d $\mathcal{N}=2$

$V_{4d} = (\phi_1 + i\phi_2, A_\mu, \psi)$ $\quad \phi_1 = A_5$

$\phi_2 = A_4$

$\rightarrow$ 2 mass defs

$\downarrow$

3d $\mathcal{N}=3$

$V_{3d} = (\phi_1 + i\phi_2 + j\phi_3, A_\mu, \psi)$ $\quad \phi_1 = A_5$

$\phi_2 = A_4$

$\phi_3 = A_3$

$\rightarrow$ 3 mass defs

2) Stress tensor multiplets:
Lorentz invariant, supersym. defs, residing in same multiplet as stress-tensor $T_{\mu\nu}$.
$\rightarrow$ unique (universal) multiplet:
- 4d $\mathcal{N}=4$ SCFTs have universal marginal def. $\mathcal{O}$ (complex, paired with its complex conjugate $\bar{\mathcal{O}}$), $\mathcal{O}, \bar{\mathcal{O}} \in [0;0]_4^{(0,0,0)}$ at top of multiplet (together with $T_{\mu\nu}$)
- 3d $\mathcal{N} \geq 4$ SCFTs contain universal relevant defs $\mathcal{M}_{\text{univ.}}$ of $\Delta = 2$ $\rightarrow$ reside at $l=2$ in stress-tensor mult. ($T_{\mu\nu}$ is at $l=4$)

Let us look at the marginal deformation in 4d $\mathcal{N}=4$ theories more closely:

$\mathcal{O}$ is exactly marginal $\rightarrow$ conformal manifold parametrized by cplx. number τ

$\rightarrow$ local geometry: $ds^2 = C \dfrac{d\tau\, d\bar{\tau}}{(\mathrm{Im}\,\tau)^2}$, $C > 0$.

τ is identified with complexified gauge coupling

The universal mass deformation $\mathcal{M}_{\mathrm{univ.}}$ in 3d $\mathcal{N}=4$:

start with $(H^i, \Psi^{i'}_\alpha)$ (free Hypermultiplet)

doublets under $SU(2)_R$ — doublets under $SU(2)'_R$

action of supercharges: $Q^{ii'}_\alpha H^j \sim \varepsilon^{ij} \Psi^{i'}_\alpha$

universal mass-def. resides in $A_2[0]^{(0;0)}_1$ at $\ell=2$

bottom component: $\varepsilon^{ij}\, \overline{H}_i H_j$.

$$\rightarrow \mathcal{M}_{\mathrm{univ}} \sim m\, \varepsilon^{\alpha\beta} \varepsilon_{i'j'}\, \overline{\Psi}^{i'}_\alpha \overline{\Psi}^{j'}$$

$$\sim m\, \varepsilon_{ij} \varepsilon_{i'j'}\, Q^{ii'}_\alpha Q^{jj'}_\beta \left(\varepsilon^{k\ell}\, \overline{H}_k H_\ell\right)$$

(L- and R-singlet)